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Preparation of entangled state of cavity QED phase matched two-photons under spontaneous parameters

Tingyu Li[1], Hui Li[1,*]

Abstract. Based on cavity quantum electrodynamics (QED), the issue of preparing entangled state of phase matched two-photon under spontaneous parameters is researched: considering the condition in which each cavity traps a two-level atom and the atom resonates and interacts with cavity field via two-photon transition process, adopting the negative eigenvalue to measure the entanglement between two subsystems and using numerical calculation to research dynamics characteristics of entanglement between two subsystems in the system. The research also discussed the influence of change of coupling coefficient between cavity fields on the entanglement characteristics between two subsystems. It is found that the entanglement between two atoms is enhanced and the entanglement between two cavity fields is weakened with increase of coupling coefficient between cavity fields.

Key words. Spontaneous parameter, Cavity quantum, Electrodynamics, Phase match, Two-photon, Entangled state.

1. Introduction

In quantum information, the information processing cannot be independent from quantum state and its evolution, while the quantum entanglement is undoubtedly the most important quantum state. As an important "quantum resource", entangled state has been widely applied in recent years with rapid development of quantum informatics, such as successfully application to domains like quantum key allocation, quantum dense coding, quantum teleportation, quantum entanglement code, and quantum computation. As the entangled state of multiple subsystems has properties not possessed by a great many two subsystems, and increasingly developing experimental technique more deepens the preparation of quantum entangled state.

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[1]School of Information Science and Engineering, Shenyang University of Technology, Shenyang, Liaoning 110870, China

*. Corresponding author

This does not relate to the essence of quantum entanglement, but also can help people to better understand the basic theories of quantum mechanics, and more importantly, can develop heaps of magical applications. The quantum entanglement is the most important and the most peculiar subject in quantum informatics.

For the issue of preparing photon entangled state, the paper proposes a preparation method for entangled state of cavity QED phase matched two-photon under spontaneous parameters: Considering the condition in which each cavity traps a two-level atom and the atom resonates and interacts with cavity field via two-photon transition process, adopting negative eigenvalue to measure the entanglement between two subsystems, and uses numerical calculation method to research dynamics characteristics of entanglement between two subsystems in the system. Lastly, the influence of change of coupling coefficient between cavity fields on the entanglement characteristics between two subsystems is discussed.

2. Definition of quantum entangled state

So-called entangled state refers to a special quantum state of the composite system. It cannot be written as direct product form of quantum state of two subsystems in any representation. To facilitate understanding, the paper considers a two-body system composed of two subsystems A and B (A and B are all of pure state), sets eigenstate vector of A as $|\Psi\rangle_A$ and eigenstate vector of B as $|\Psi\rangle_B$. If the eigenstate vector $|\Psi\rangle_{AB}$ of the composite system A+B cannot be expressed as direct product form of $|\Psi\rangle_A$ and $|\Psi\rangle_B$, the pure state $|\Psi\rangle_{AB}$ is called an entangled state.

$$|\Psi\rangle_{AB} \neq |\Psi\rangle_A \otimes |\Psi\rangle_B \ . \tag{1}$$

When mixed state is considered, the density matrix can be used for representation,

$$\rho_{AB} \neq \rho_A \otimes \rho_B \ . \tag{2}$$

$$|\Psi\rangle_{AB} = \alpha |0\rangle_A |0\rangle_B + \beta |1\rangle_A |1\rangle_B, \quad |a|^2 + |\beta|^2 = 1, \tag{3}$$

is an entangled state in condition of pure state.

Following takes the maximal entangled state of two particle systems whose spin is respectively $\frac{1}{2}$-Bell base for example to describe the definition of entangled state. There are following four quantum states for two two-particle quantum systems, i.e. the eigenstate of Bell operator:

$$\left|\Phi^{\pm}\right\rangle_{12} = \frac{1}{\sqrt{2}} \left(|0\rangle_1 |0\rangle_2 \pm |1\rangle_1 |1\rangle_2\right) \ . \tag{4}$$

$$\left|\Psi^{\pm}\right\rangle_{12} = \frac{1}{\sqrt{2}} \left(|0\rangle_1 |1\rangle_2 \pm |1\rangle_1 |0\rangle_2\right) \ . \tag{5}$$

Assuming there are two atoms 1 and 2 with only two quantum states, which can be in superposition state-one of the states in equation (4b), $|\Psi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|0\rangle_1 |1\rangle_2 -$

$|1\rangle_1 |0\rangle_2)$, where, $|0\rangle_1 |1\rangle_2$ represents the atom 1 is in state $|0\rangle$ and atom 2 is in state $|1\rangle$. $|1\rangle_1 |0\rangle_2$ represents atom 1 is in state $|1\rangle$, and atom 2 is in $|0\rangle$. When the two atoms are in superposition state $|\Psi^\pm\rangle$, the two atoms are regarded as in entangled state, as now it is only known that one atom is in $|0\rangle$ state and one atom is in state $|1\rangle$, but which atom is in state $|0\rangle$ and which atom is in state $|1\rangle$ are unknown. The atom 1 may be in state $|0\rangle$ or state $|1\rangle$, and atom 2 may be in state $|0\rangle$ or state $|1\rangle$. Thus, the two atoms are entangled.

As each component of entangled state is composed of singlet state $|0\rangle$ and $|1\rangle$ of two particles, so the two particles in entangled state $|0\rangle |1\rangle$ have a wonderful characteristic: once the state $|0\rangle$ of the first particle is determined via measurement, the corresponding wave function of entangled state collapses to its corresponding component $|1\rangle$, thereby determining the state $|1\rangle$ of another particle in an instant. Now even the spatial distance between two particles is far (several meters, thousand of meters, or tens of thousands meters), people can determine the state of one particle through the state of another particle in an instant in principle. For example, for a two-atom system in state $|0\rangle |1\rangle$, if measurement of atom 1 finds that it is in $|0\rangle$ state, then people can immediately know atom 2 is in $|1\rangle$ state, which is called by Einstein as "spooky interaction between places with far distance"

3. Preparation of entangled state

The quantum entangled state plays a vitally important role in quantum teleportation, quantum dense coding, quantum cryptographic communication and quantum calculation, so the preparation of quantum entangled state is key in the quantum information domain. To entangle two or more microscopic systems in direct product state requires controllable interaction. Currently, the preparation of entangled state has been realized in some physical systems, for example: nonlinear optical system, cavity quantum electrodynamics (QED) system, ion trap system and recently realized entanglement of atom cluster. Currently, for the two-particle system, the most successful achievement is to use spontaneous parametric down-conversion to realize two-photon entanglement in nonlinear optical system. Following is a brief introduction of the schemes of preparing photon entanglement via spontaneous parametric down-conversion and preparing atom entanglement in cavity QED, and preparing entangled state in ion trap.

3.1. Preparation of entangled state by spontaneous parametric down-conversion

Spontaneous parametric down-conversion is the non-linear interaction process of crystal, and the intensity of non-linear interaction is determined by electrochemical intensity of nonlinear crystal:

$$p_i = \sum X_{ij} \, E_j \left(\omega_1\right) + \sum X_{ijk}^{(2)} \, E_j \left(\omega_1\right) E_k \left(\omega_2\right)$$
$$+ \sum X_{ijkl}^{(3)} \, E_j \left(\omega_1\right) E_k \left(\omega_2\right) E_j \left(\omega_2\right) E_l \left(\omega_3\right) + \cdots \cdots$$

Where, the conversion of parameters is produced by intermediate second-order non-linear interaction, and the interaction of higher-order terms is very small and negligible. This process must meet law of conservation of energy, i.e.:

$$\omega_p = \omega_s + \omega_i, \quad \vec{k}_p = \vec{k}_i + \vec{k}_s \ . \tag{6}$$

This equation is also called phase matching condition, ω_p, and $\vec{k}_p, \vec{k}_s, \vec{k}_i$ respectively represent frequency and wave vector of pump light, signal light and leisure light. The birefringence of crystal leads to different refractive indices of different polarized light in crystal, and the dispersion action of crystal can realize phase matching in some crystals, so proper nonlinear crystals can be selected to realize spontaneous parametric down-conversion.

3.2.   *Preparation of two-photon entangled state*

The research used spontaneous parametric down-conversion process of continuous wave laser beam pump nonlinear crystal to prepare two-photon polarization-entangled state: radiate a beam of pump light on a nonlinear crystal BBO to produce a pair of entangled twin photons. The spontaneous parametric down-conversion is the non-linear interaction process of crystal, and can be divided into type I and type II according to the type of phase matching of crystal. Following is an introduction of the characteristics of the two types (Take uniaxial negative crystal as an example)

The type I parametric down-conversion process can be expressed as $e \to o + o$, i.e. the two photons produced have the same polarization and are all vertical to polarization direction of pump light. The spatial distribution of parametric light produced is coniform with pump light as axis, as shown in Fig.1:
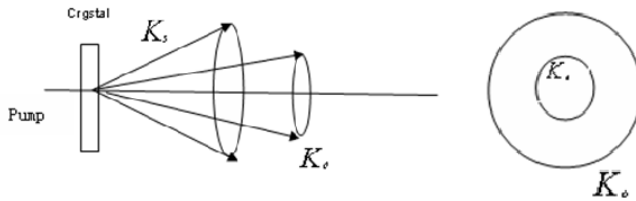

Fig. 1. Type I spontaneous parametric down-conversion

This type produces two-photon state entangled in time, space and frequency. Type II parametric down-conversion can be expressed as $e \to e + o$, i.e. the two photons produced are mutually perpendicular in polarization direction. It usually adopts frequency degeneration, now the polarization-entangled two-photon pairs can be produced, as shown in Fig.2

The distribution of parametric light includes two tapers in the case of nonlinear matching. In the figure, the upper half circle is e light and the lower half circle is o light, whose two crossing points may be e light or o light. In this way, a pair of photons in these two directions forms polarization-entangled photon state. In 1999, Kwait, et al. suggested a new method of producing polarization-entangled twin
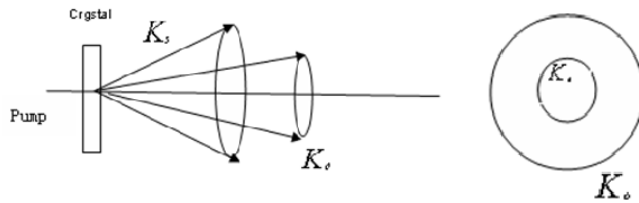
Fig. 2. Type II parametric down-conversion

photons, i.e. adopted type I nonlinearly phase-matched BBO crystal, when the two crystals were bonded, the optical axis of two crystals are in two orthogonal planes, when a beam of polarized pump light is radiated into this interwoven crystal, a polarization-entangled twin photons are produced. A large advantage of this method is to conveniently produce non-maximally entangled state by just changing polarized state of pump light.

The entanglement source brightness and entanglement degree of this preparation method are all close to leading level of international like research. Besides, our entanglement source is also featured with adjustability of parameter, i.e. it cannot only produce the common maximal entangled state, but also can conveniently produce non-maximal entangled states of various entanglement degrees, with entanglement degree easy for control. This provides a powerful and convenient tool for research on change of various properties of entangled state. We also used this kind of entanglement source to prepare another important mixed entangled state in quantum informatics-Werner state. The scheme adopted made the entangled components in Werner state controllable. The Werner state can be directly used for experimental study on entanglement purification, which is of great importance for theoretical research, experimental research or even research on practicability for quantum communication.

3.3.   Prepare atom entangled state in cavity QED

A two-level atom is equal to a particle whose spin is $\frac{1}{2}$, and the detection efficiency for atom can be basically reach 100%. Furthermore, the atom is highly spatially separable. Thus, to prepare entangled state in cavity QED is a significant subject.

In cavity QED, quantum state evolution of atom—optical cavity system can be described with Jaynes—Cummings model. Phoenix, et al., Kudryavtseu, et al., and Cirac et al., respectively suggested two-atom EPR state. In 1997, the research group of Ecole Normale Supérieure successfully prepared EPR state in experiment by injecting an atom which is initially in excited state $|e\rangle$ into an optical cavity which is initially vacuum, and after $\frac{\pi}{2}$ Rabi rotation, following is obtained:

$$|\Psi\rangle_1 = \frac{1}{\sqrt{2}} \left(|e,0\rangle + |g,1\rangle \right) \tag{7}$$

To read out state of light field requires an atom in ground state $|g\rangle$ to enter the optical cavity to be subjected to Rabi rotation with phase of π, then the two atoms

are in following entangled state:

$$|\Psi\rangle_2 = \frac{1}{\sqrt{2}} \left(|e,g\rangle - |g,e\rangle\right) . \tag{8}$$

Above methods were produced when atom and optical field resonate and interact. In 2000, Zheng and Guo suggested directly injecting two two-level atoms into a non-resonant cavity field to prepare two-atom entangled stage. Then the research group of Ecole Normale Supérieure successfully applied this theoretical scheme to experiment. What follows was popularization of this theory to preparation of multiple atoms entangled state.

4. Experimental analysis

4.1. Experiment setup

The experiment was in three parts:

1) Part 1 (built light path, as shown in Fig.3), firstly, according to the core map for the principle part of this application form, referred to the parameters of each device provided by laboratory, made theoretical calculation, prejudged pattern of SP-DC optical field and spatial position of entangled point, designed and planned reverse lighting path system, pre-set positions of devices. Then, used high-resolution spectrograph and large-travel precision displacement instrument to look for approximate position of entangled point, and further collected and analyzed the correlation of spectral information and intensity of SPDC optical field near entangled point with positional change, and measured the spatial position curve of light with wavelength of 806nm near entangled point, then described spatial distribution in the condition of SPDC frequency degeneration using curve shape, to promote understanding the generation mechanism of optical field. This part also serves as the instructive technology index for the experiment in part 2.
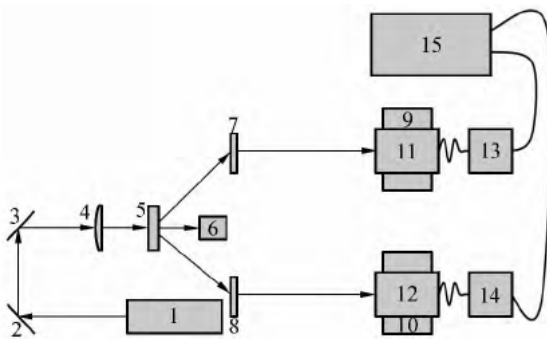

Fig. 3. Analytic system diagram of SPDC optical field

[1. Laser 2,3. Visible light reflector 4. Focusing lens 5. Main BBO crystal 6. Tail light collecting vessel 7,8. Infrared reflector 9,10. Large travel travelling platform

11,12. Optical fiber collimator 13,14 Spectrograph 15. Computer system (configured with spectrograph collection control software)

2) Part 2. Based on part 1, connected the received optical signal into single photon counter near entangled point, measured decay curve of single-channel counting and took it as area coverage information for looking for spatial positions of entangled point. This part also provides data technology direction for building entanglement source in part 3. .

3) Part 3, as shown in Fig.4, built light path, added analyzer via twin channels to analyze data change to precisely determine the position of entangled point, then disconnected spectrograph, and connected the signal to single photon counter-electronic screening and coincidence analytic system, added compensation, made slight adjustment. Completed overall experimental process of building entanglement source, collected data and drew up coincidence counting analytical curve, substituted the key data into exporting form of Bell inequation-CHSH inequation, and tested calculation result of inequation, thereby judging whether entanglement source was successfully built and the receiving efficiency of entanglement source was high or low, and verifying basic concepts of quantum mechanics, to achieve the set target of experiment.
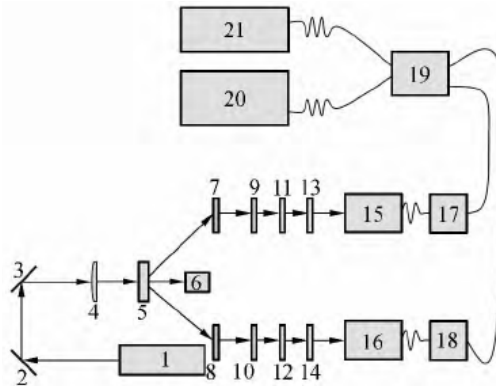

Fig. 4. Performance testing system diagram for entanglement source

[1. Laser 2,3. Visible light reflector 4. Focusing lens 5. Main BBO crystal 6. Tail light collecting vessel 7,8. Infrared reflector 9,10. Broadband half-wave plate 11,12. Compensating BBO crystal; 13,14 Analyzer 15. Monomode optical fiber collimator 17,18. Single photon counter 19. Electronics system 20. Computer system 21.Oscillograph system]

4.2. Result analysis

As shown in Fig.3, configured light path and device, collected down-conversion spatial map at 806nm near the entangled points, as shown in Fig.5. The curve result in the figure clearly shows the condition in the neighborhood of intersecting point for 806 nm aureole. According to the principle of this experiment, the intersecting point is just the entangled point, thereby completing the analysis on focused SPDC

optical field, and then assisted looking for the preliminary positioning for inter-umbrella position of entangled point.
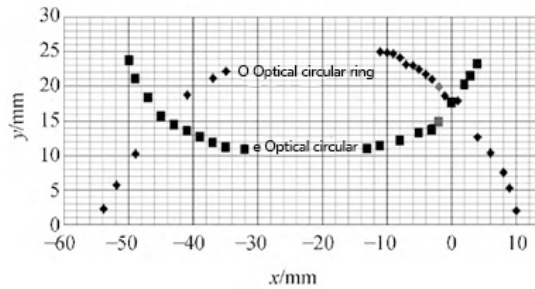

Fig. 5. Space map of down-conversion at 806nm near entangled point

Converted the spectrograph system in Fig.4 into single photon counter, collected decay pattern of transverse and longitudinal counting rate near entangled point, as shown in Fig.6 (background noise grade was 1,400). The curve in the figure shows descending condition of single-channel data near the entangled point, and it can be judged that the adjustable scope from preliminary positioning to pinpointing is about 0.2mmX0.2mm. Besides, observation of the curve shows that outside the aureole where the entangled point is located is sparse area of photons.
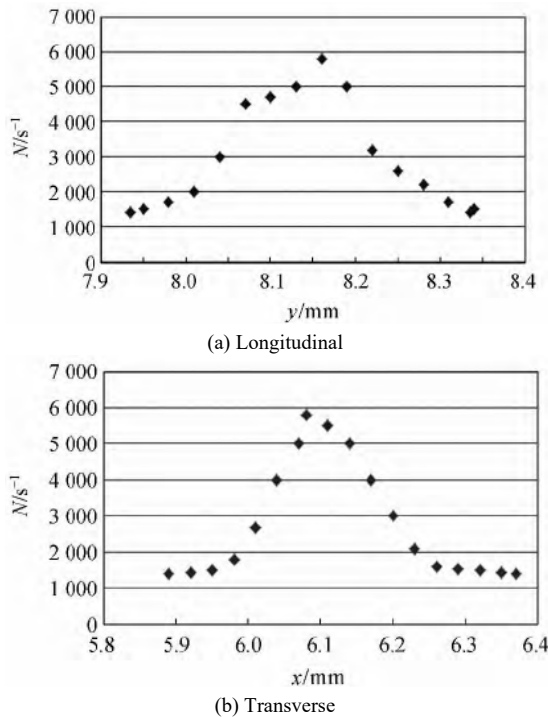

(a) Longitudinal

(b) Transverse

Fig. 6. Decaying of longitudinal and transverse counting rate near entangled point

General analysis on experimental data result: The analysis on SPDL optical field visually and vividly presents the spatial position of entangled point, and further shows the change of strength of photons near entangled point, thereby conducing to deepen the understanding of down-conversion optical field and technological details for spatial positioning of entangled points, also realizing orderly and quick next precision spatial positioning of entangled point The entanglement source was successfully built via this method with good quality delivered, which speaks volumes for the fact that the experimental method perfectly kept original technical quality of entanglement source while optimizing, thereby proving the success of this method.

5. Conclusion

By measuring SPDC optical field near entangled point with high-sensitivity spectrograph, the paper researches the preparation of entangled state of phase matched two-photon under spontaneous parameters, and compares with theoretical result, thereby deepening the cognition of formation mechanism of SPDC optical field and promoting understanding of the concept of quantum entanglement and experimental phenomenon. Lastly it uses numerical calculation method to research dynamics characteristics of entanglement between two subsystems in the system and discusses the influence of change of coupling coefficient between cavity fields on the entanglement characteristics between two subsystems, and further applies this data result to assist positioning entangled point's position in optical field to assist building two-photon polarized entanglement source, thereby reducing the operational difficulty in such traditional entanglement source experiment and shortening the time needed to complete entanglement source experiment.

References

[1] N. ARUNKUMAR, S. JAYALALITHA, S. DINESH, A. VENUGOPAL, D. SEKAR: *Sample entropy based ayurvedic pulse diagnosis for diabetics.* IEEE-International Conference on Advances in Engineering, Science and Management, ICAESM-2012, art. No. 6215973 (2012), 61–62.

[2] Y. J. ZHAO, Y. H. HU, J. J. LIU: *Random Triggering-Based Sub-Nyquist Sampling System for Sparse Multiband Signal.* IEEE Transactions on Instrumentation and Measurement *66* (2017), No. 7, 1789–1797.

[3] X. DU, L. CHEN, D. HUANG, Z. PENG, C. ZHAO, Y. ZHANG, Y. ZHU, Z. WANG, X. LI, G. LIU: *Elevated Apoptosis in the Liver of Dairy Cows with Ketosis.* Cellular Physiology & Biochemistry *43* (2017b), No. 2, 568–578.

[4] N. ARUNKUMAR, K. RAM KUMAR, V. VENKATARAMAN: *Automatic detection of epileptic seizures using permutation entropy, Tsallis entropy and Kolmogorov complexity.* Journal of Medical Imaging and Health Informatics *6* (2016), No. 2, 526–531.

[5] Y. Y. ZHANG, A. ALGBURI, N. WANG, V. KHOLODOVYCH, D. O. OH, M. CHIKINDAS, AND K. E. UHRICH: *Self-assembled Cationic Amphiphiles as Antimicrobial Peptides Mimics: Role of Hydrophobicity, Linkage Type, and Assembly State*, Nanomedicine: Nanotechnology, Biology and Medicine *13* (2017), No. 2, 343–352.

[6] Y. SONG, N. LI, J. GU, S. FU, Z. PENG, C. ZHAO, Y. ZHANG, X. LI, Z. WANG,

X. Li: *β-Hydroxybutyrate induces bovine hepatocyte apoptosis via an ROS-p38 signaling pathway.* Journal of Dairy Science *99* (2016), No. 11, 9184–9198.

[7] N. Arunkumar, K. R. Kumar, V. Venkataraman: *Automatic detection of epileptic seizures using new entropy measures.* Journal of Medical Imaging and Health Informatics *6* (2016), No. 3, 724–730.

[8] R. Hamza,   K. Muhammad,   N. Arunkumar,   G. R. González: *Hash based Encryption for Keyframes of Diagnostic Hysteroscopy*, IEEE Access, https: //doi.org/10.1109/ACCESS.2017.2762405 (2017).

[9] J. W. Chan, Y. Y. Zhang, and K. E. Uhrich: *Amphiphilic Macromolecule Self-Assembled Monolayers Suppress Smooth Muscle Cell Proliferation*, Bioconjugate Chemistry *26* (2015), No. 7, 1359–1369.

[10] Y. J. Zhao, L. Wang, H. J. Wang, and C. J. Liu: *Minimum Rate Sampling and Spectrum Blind Reconstruction in Random Equivalent Sampling.* Circuits Systems and Signal Processing *34* (2015), No. 8, 2667–2680.

[11] S. L. Fernandes, V. P. Gurupur, N. R. Sunder, N. Arunkumar, S. Kadry: *A novel nonintrusive decision support approach for heart rate measurement.* Pattern Recognition Letters. https: //doi.org/10.1016/j.patrec.2017.07.002 (2017).

[12] N. Arunkumar,   K. Ramkumar,   V. Venkatraman,   E. Abdulhay, S. L. Fernandes, S. Kadry, S. Segal: *Classification of focal and non focal EEG using entropies.* Pattern Recognition Letters *94* (2017), 112–117.

[13] J. W. Chan, Y. Y. Zhang, and K. E. Uhrich: *Amphiphilic Macromolecule Self-Assembled Monolayers Suppress Smooth Muscle Cell Proliferation*, Bioconjugate Chemistry *26* (2015) No. 7, 1359–1369.

[14] M. P. Malarkodi, N. Arunkumar, V. Venkataraman: *Gabor wavelet based approach for face recognition.* International Journal of Applied Engineering Research *8* (2013), No. 15, 1831–1840.